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Heat Conduction in Two-Phase Heterogeneous Materials: Analytical Study

The method of the calculation of the heat conduction of the two-phase heterogeneous materials, where the filler's participles of the cubic form by the at random fashion distributed in the matrix's volume is considered. The given method of the effective heat conduction of the heterogeneous materials is based on the probability principles of the analysis of their structure. Such approach allows not only predict the average magnitudes of the heat conduction of the binary or many-component compositions but also to determine the bounds where their measured valuations will be realized with the given of the degree of certainty. The received formulas are applied in a wide range volume correlation changes and heat conduction of the separated components and also typical sizes of the researched samples.

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1. Introduction

Heterogeneous propellant combustion is a very complicated process. The complexity starts with the material itself. Composite propellants are a mix of crystalline oxidizer and polymer binder, and have heterogeneous length scales on the order of 100 μm or more. When these materials are burned, flames with characteristics of both premixed flames and diffusion flames can form in the gas phase [1-5]. The surface of the burning propellant is multi-phase and three-dimensional, resembling a boulder field on which molten binder can flow. It is important to realize that significant chemistry is occurring in the gas phase, in the solid phase, and at the surface. When materials such as aluminum are added to the mix, the complexity increases by another order of magnitude. It is safe to say that all models previously developed or currently in development account for only a fraction of these phenomena [6-10].

The ability to accurately measure the thermal conductivity of insulation materials is critical to the establishment, assurance and enforcement of thermal performance standards. There is little point in devising parameters for quality assurance if compliance by quantitative measurement cannot be established. Thermal conductivity apparatuses are notorious for systematic errors and their performance should be verified over the intended temperature range of measurement on appropriate 'standard' materials [11-15]. The experimental design for all the thermal conductivity values presented in the literature have been based on the assumption the Fourier equation for heat conduction in a solid is valid. Unfortunately this experimental design is flawed because it neglects the effect of the temperature dependence of the thermal conductivity. Neglecting the temperature dependence of the thermal parameters introduces systematic errors into, for example, the estimated thermal conductivity values. These systematic errors are typically of the order of 5% to 20% in typical heat flow in buildings experiments. In recent years these systematic effects have become large when compared to the experimental uncertainties. Obviously such systematic effects will effect and undermine the currently adopted national and international standards. It is shown in this project that acceptably accurate information can be achieved by improving the measurement and analysis procedures that are used to determine the thermal properties of solid materials [16-20].

Thermal conduction measurement techniques rely on the direct measurement of the heat that passes through the material and the temperature at different positions in the material. The techniques also require thermometers to measure the temperature and its derivatives in the material as they change with time. A number of steps were taken in this project to ensure that the data collected were of high quality. The temperature dependence of the electrical resistance signal cables was reduced by employing compensation cables and the thermal stability of these cables was improved by surrounding them with several layers of Aluminised Mylar. Faraday cages were used to minimise the influence of external factors on the signals. Earth loop and mains noise were largely eliminated where possible by using battery power for the electronics, logging equipment and signal amplifiers and 1000Ω Platinum resistance thermometers were used in preference to the usual 100Ω thermometers. A temperature difference sensor has been developed by RAL during the project with an appropriate calibration apparatus that has a repeatable precision of better than 0.200 mK. In order to get the best accuracy when installing the thermometer in an experiment, it is essential that the calibration be made in conditions as close as possible to those go like it to be experienced by the sensors. The experimental design of the prototype has ensured that the uncertainty in the variables such as linear dimension, positioning of sensors and the error due to the radial loss of heat flux are all much less than 1%. The dominant source of uncertainty is in the measurement of the power entering the radiation cavity at the top of the bar or the measured heat flux that propagates along the bar. The apparatus has been designed such that they heat flux of 1W m^{-2} will result in a temperature difference of 1 mK [21-25].

2. Analysis and Treatment

Let's consider a binary system with constant values of the heat conduction of the components λ_1 (matrix) и λ_2 (filler). We usually get the design equations for heat conduction λ of the composition systems by dividing material with the help of the adiabatic or isothermic surfaces [1]. Let's denote the received comparative valuations of the effective heat conduction compositions γ′ and γ″ (γ′, γ″ = λ_{eff}/λ_1) respectively. The calculated values for γ′ and γ″ go with experimental data if the filler directed in the matrix length and breadth of direction distribution heat. It's better to find the effective heat conduction in the form of linear aggregate γ′ and γ″ for the systems with chaotic disposition of the participles filler:

$$\gamma_{eff} \equiv \frac{\lambda_{eff}}{\lambda_1} = A\gamma' + B\gamma'' \qquad (1)$$

where A and B the numbers, satisfying the proportion A + B = 1.

The coefficients A and B characterize isotherms distortion near participles of the filler. They usually suppose in deriving design formula, that lines of flow (of the heat flow) are parallel. However, particularly, if $\lambda_2/\lambda_1 \equiv \nu \to \infty$ almost all lines of flow pass through the filler. In this case the ratio of the lines of flow by the considering cross-sectional to the general numbers of the lines, passing through the surface, which is perpendicular to the heat flow with the same its density, equals volumetric degree of the P filler. In case that ν = 0, this proportion equals 1 – P. If 0 < ν < ∞ the distortion of the lines of flow determine not only by the filler degree but also by the thermal and physical characteristics of the material of the filler and the matrix, character of the heat exchange on their bounds and etc. If we consider these factors, it'll result in complication of the considering problem. So then we'll adopt the assumption that with all ν > 1 in the formula (1) B = P, A = 1 – P, and with all ν < 1 B = 1 – P, A = P. Here we, of course, narrow, in some cases, the bounds of the got below formulas [16-18].

Let's write the proportion for the effective heat conduction λ or the heterogeneity element by emphasizing the heterogeneity element with adiabatic surfaces:

$$\frac{\delta+\beta}{\lambda} = \frac{\delta}{\lambda_1} + \frac{\beta}{\lambda_2} \qquad (2)$$

where by putting signs $\lambda / \lambda_1 \equiv \gamma$, we'll get:

$$\gamma = \frac{\delta+\beta}{\delta+\dfrac{\beta}{\nu}} \qquad (3)$$

Let's use the above got formula for the joint density to find density of distribution of the chance function γ:

$$f(\beta,\delta) = \frac{\ln P \cdot \ln(1-P)}{\alpha P} \cdot \frac{1}{\beta} P^{\beta/\alpha}(1-P)^{\delta/\beta} \qquad (4)$$

The distribution function of the comparative heat conduction γ, the proportion will be determined:

$$F(\gamma) = \int_0^\infty \left[\int_a^b f(\beta,\delta)d\delta \right] d\beta \qquad (5)$$

Here we have the limit of integration a and b, which we can determine from the proportion (3), depending on ν the different values are adopted:

if ν<1 a=0, $b = \frac{\beta}{\nu} \cdot \frac{\gamma-\nu}{1-\gamma}$;

if ν>1 $a = \frac{\beta}{\nu} \cdot \frac{\nu-\gamma}{\gamma-1}$, b=+∞

If we integrate we'll get:

$$F(\gamma) = 1-(1-P)^{\frac{1}{\nu}\cdot\frac{\gamma-\nu}{1-\gamma}} \qquad (\nu < 1) \quad (6)$$

$$F(\gamma) = (1-P)^{\frac{1}{\nu}\cdot\frac{\gamma-\nu}{1-\gamma}} \qquad (\nu>1) \quad (7)$$

The differentiation of the (6) and (7) result in density distribution expressions f(γ), and differ only by multipliers: 1 - ν if ν < 1 and ν - 1 if ν > 1. If we combine these expressions into one, we'll the following density distribution γ:

$$f_1(\gamma) = -\frac{|\nu-1|}{\nu} \cdot \frac{\ln(1-P)}{(1-\gamma)^2} \cdot (1-P)^{\frac{1}{\nu}\cdot\frac{\gamma-\nu}{1-\gamma}} \qquad (8)$$

The central tendency of the comparative heat conduction γ′ will determine by the proportions:

$$\gamma' = m_\gamma = \int_1^\nu \gamma f(\gamma) d\gamma \qquad (\nu>1) \quad (9)$$

$$\gamma' = m_\gamma = \int_\nu^1 \gamma f(\gamma) d\gamma \qquad (\nu < 1) \quad (10)$$

If we integrate (9) and (10), we'll get formula for γ′, true for any ν:

$$\gamma' = 1 - \frac{1-\nu}{\nu}\left((1-P)^{-\frac{1}{\nu}} \ln(1-P) \cdot Ei\left[\frac{\ln(1-P)}{\nu}\right]\right. \quad (11)$$

where

$$Ei(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$$

is an integral exponent

We should cut the material with the surface to find γ'', which is perpendicular to the heat flow and isolate in this surface an area S in n times larger of the sectional area of the one participle of the filler ($S = n\alpha^2$). The possibility that cross-sectional S hits exactly m participles of the filler equals:

$$F_m = C_n^m P^m (1-P)^{n-m} \quad (12)$$

The possibility that all square will be filled with filler (m = n) equals P^m. If we continue to reason further as we did in our work [2], we'll get the similar expression by the structure for density of distribution of the squares S_2, filled with filler phase to get the expression for density of distribution of the length continuous phase filler:

$$f(S_2) = -\frac{\ln P}{\alpha^2} P^{S_2/\alpha^2} \quad (13)$$

We find in the same way density of distribution of the squares S_2, filled with matrix material:

$$f((S_1) = -\frac{\ln(1-P)}{\alpha^2}(1-P)^{S_1/\alpha^2} \quad (14)$$

If we isolate the layer with the thickness Δx in the material, which is set between two isothermic surfaces, we'll get the following expression for effective heat conduction λ determination:

$$\lambda \frac{\Delta T}{\Delta x} S = \lambda_1 \frac{\Delta T}{\Delta x} S_1 + \lambda_2 \frac{\Delta T}{\Delta x} S_2 \quad (15)$$

where ΔT is temperature difference in the selected layer.

Then if we put the symbols

$$q = \frac{S_2}{S_1}$$

and considering, that $S_1+S_2=S$, we'll get the following expression for relative heat conduction of the heterogeneity element by dividing material with the help of isothermic surfaces:

$$\gamma = 1-(1-\nu)\frac{q}{1+q} = \varphi(q) \quad (16)$$

where

$$q = \frac{\gamma-1}{\nu-\gamma} = \psi(\gamma) \quad (17)$$

Let's first find distribution function F(q):

$$F(q) = \int_0^{\infty} f(S_1)\left[\int_0^{qS_1} f(S_2)dS_2\right]dS_1 \quad (18)$$

We'll get by putting (18) into expressions (13) and (14):

$$F(q) = 1 - \frac{\ln(1-P)}{\ln(1-P) + q \cdot \ln P} \quad (19)$$

We'll find density of distribution f(q) by differentiate:

$$f(q) = \frac{\ln(1-P) \cdot \ln P}{[\ln(1-P) + q \cdot \ln P]^2} \quad (20)$$

As consistent with [3]:

$$f(\gamma) = \varphi[\psi(\gamma)] \cdot |\psi'(\gamma)|$$

We can find density of distribution considering (16) and (17):

$$f_2(\gamma) = \frac{|\nu-1|\Phi(P)}{\{\nu - \Phi(P) + \gamma[\Phi(P)-1]\}^2} \quad (21)$$

where

$$\Phi(P) = \frac{\ln P}{\ln(1-P)}$$

We can determine the central tendency of the relative heat conduction γ'' by the formulas similar to (9) and (10). We'll get by integrating:

$$\gamma'' = \frac{\nu - \Phi(P)}{1-\Phi(P)} + \frac{(\nu-1)\Phi(P)}{[1-\Phi(P)]^2}\ln \Phi(P) \quad (22)$$

Considering the proportions (11) and (22) and also the above made assumption concerning coefficients A and B in the formula (1), we can write the final expression for central tendency of the heat conduction of the heterogeneous material; specifically if $\nu > 1$:

$$\gamma_{eff} = (1-P)\cdot\left\{1 - \frac{1-\nu}{\nu}\left((1-P)^{-\frac{1}{\nu}}\ln(1-P)\cdot Ei\left[\frac{\ln(1-P)}{\nu}\right]\right)\right\} + P\cdot\left\{\frac{\nu-\Phi(P)}{1-\Phi(P)} + \frac{(\nu-1)\Phi(P)}{[1-\Phi(P)]^2}\ln\Phi(P)\right\} \quad (23)$$

With the made assumptions, the effective heat conduction doesn't depend on the participle size in the filler. The dependence γ_{eff} from P with different ν is shown in the Fig. (1).

The estimated value of the difference meanings of the heat conduction of the samples is a great interest of practical applications. We'll get the following expression for density of distribution of the relative heat conduction with the use material dividing circuit by adiabatic surfaces if we summarize the used binominal distribution in case of arbitrary parameter $n=L/\alpha$ (where L is sample's thickness towards heat flow) in the work [2].

$$\varphi(\gamma) = \frac{\nu\Gamma(n+1)P^{Cn}(1-P)^{n(1-C)}}{\gamma^2(\nu-1)\Gamma(Cn+1)\Gamma(n-Cn+1)} \quad (24)$$

where

$$C = \frac{\nu(\gamma-1)}{\gamma(\nu-1)}$$

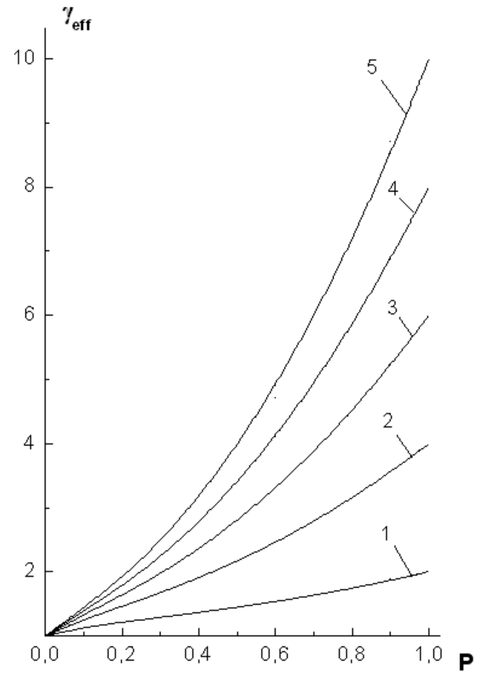

Fig. (1) The dependence of the relative effective heat conduction from the volumetric degree of the filling, calculated by formula (23) with heat conduction proportions of the filler and matrix ν = 2(1), 4(2), 6(3), 8(4) and 10 (5)

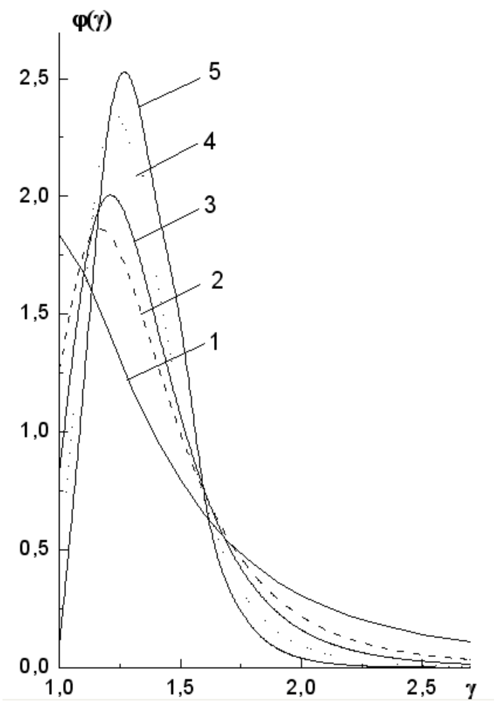

Fig. (2) The density of distribution of the relative heat conduction, calculated by formula (24) if proportion valuations of the sample's thickness to the size of the participle filler n = 2(1), 5(2), 7(3), 10(4) and 15(5)

The dependences with φ(γ) different n for the case *P*=0.3 and *ν*=5 are shown in the Fig. (2). If we know φ(γ), the possibilities of getting heat conduction valuations, lying inside of the fixed bounds will be easy to calculate.

The dependence form parameter k of the possibility V of the receiving relative heat conduction valuations, which is differing from average of distribution (γ_0) to the value not more ± $k\gamma_0$:

$$V = \int_{\gamma_0(1-k)}^{\gamma_0(1+k)} \varphi(\gamma)d\gamma \qquad (25)$$

The calculation by the formula (25) is made for P=0.7 and different n and ν. As it was expected, the value V is strongly decreasing with the physical heterogeneity material increase (ν) and material thickness decrease (L), as in Fig. (3).

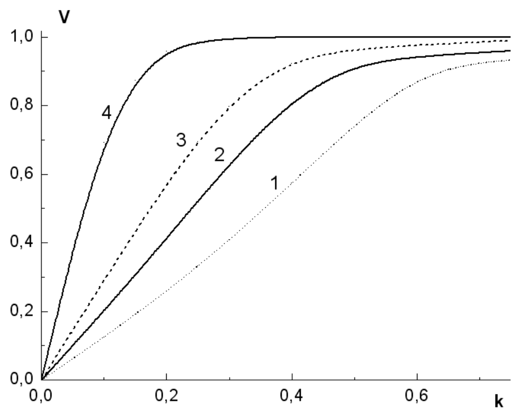

Fig. (3) The possibility of the getting valuations of the relative heat conduction, which is differing from the average of distribution (γ_0) to the value not more ± $k\gamma_0$ with P = 0,7 and different n and ν:
(1) n = 5.75, ν = 10; (2) n = 23, ν = 100; (3) n = 23, ν = 10; (4) n = 230, ν = 10

Conclusion

It's enough to calculate the effective heat conduction of the one heterogeneity element to find the effective heat conduction composition. Let, as in [2], δ is a continuous length phase of the matrix, β is a length phase of the filler, z=δ+β is an element's length of the heterogeneity. The filler consists of the cubes with the size of the edge α, the bounds of which are reciprocally parallel; the heat flow is directed perpendicular to one of the bounds.

The received dependences allow us to find the limits, where heat conduction of the heterogeneous two-phase materials can vary depend upon sample's thickness, filler degree and heat conduction components.

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Introduction to Autonomous Mobile Robots

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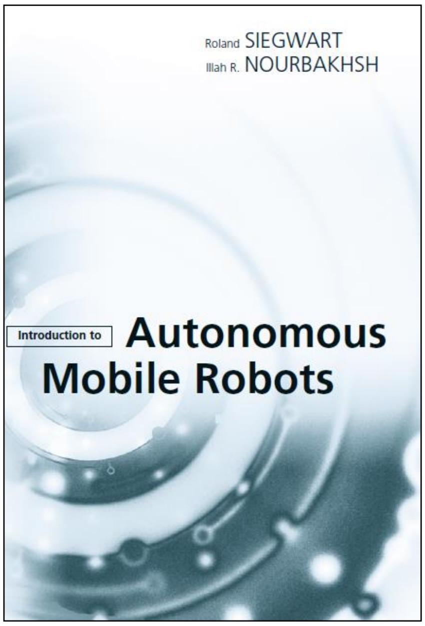

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