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Simulation of Positive Streamer Discharge in a 0.5 cm Air Gap

The aim of the work is to simulate the dynamic of positive streamer in a plane-to-plane gap filled with air at 300K and atmospheric pressure. A simulation was performed using 2-dimensional axisymmetric fluid model. The model was constructed using the finite element method (FEM) and a time-dependent solver in the commercial computer package COMSOL Multiphysics version 6.0. Ionization by electron impact, attachment, and recombination are among the chemical reactions taken into account in this model. By solving the drift-diffusion equations for ions and electrons in conjunction with Poisson's equation, the mobility of charged species is simulated. Modeling results showed that (i) the strong electric field surrounding the streamer's head cause a notable enlargement of the streamer head. The electric field values needed to bridge the gap were E_{br}=32 kV/cm (ii) the positive streamer's particle density falls between 10^{13}-10^{14} cm^{-3} and the breakdown time for the short gaps occurs in several nanoseconds.

Keywords: Streamer discharge; Hydrodynamic model; Finite element method
Received: 26 November 2023; **Revised:** 26 January; **Accepted:** 02 January 2024

1. Introduction

In many applications, atmospheric air serves as the most popular medium for electrical insulation [1]. A non-conducting substance can become conducting through a process known as electric breakdown [2]. When an insulating material, like air, is subjected to a high enough electric field, an electric discharge happens [3]. To initiate electron avalanches, the 'applied field' must be larger than a threshold value known as the conventional breakdown threshold field E_k, which is defined by the equality of the dissociative attachment & ionization coefficients. It is about 3.2x10^6 V/m at 'ground pressure' in air. The space charge field equals the applied field when the number of free electrons in an electron avalanche reaches a critical amount (~10^8 at ground pressure). The avalanche has changed into a streamer at this point, and the space charge field is mostly in charge of regulating the discharge dynamics [4,5]. Since streamer discharges were the 'first stage' of electric breakdown, they have been the subject of experimental research for almost a century [6]. Streamers are commonly used in high-voltage technology and cold atmospheric plasma applications [7]. The real plasma channel is generated by a channel of ions left behind by a small area of high electron concentration, known as the streamer head, as it moves toward an electrode [8] where the field is very low ($E \leq E_k$) [9]. Filamentary discharges often occur at high values of the gap length multiplied by the gas pressure, $pd \geq$(1-3) atm.mm [10]. Physically, the streamer's head is the most significant area since it produces a strong field there because of the separation of charges. Species created in the streamer head cause chemical reactions in a gas. The head's electrons are moved in relation to the ions, and the polarization field that results makes it easier for additional electrons to be produced, which both excite and destroy molecules. The streamer head's electron energy of roughly 10 eV is ideal for molecular excitation and dissociation. The streamer head emits both excited species and atoms while it moves [11].

In many natural occurrences, there are streamers of both polarities [12]. In plane-plane geometry [13], based on how the streamer propagates in relation to the electric field within each of these channels [14]. Positive or negative polarity can be found in streamers. While negative streamers propagate in the opposite direction from the applied electric field, positive streamers propagate in the direction of the electric field [15]. Double-headed streamers are another type that spread in both directions [16]. For the negative surface streamer, the maximal electric field is about 20-25 kV/mm; for the positive surface streamer, it is over 30 kV/mm [17]. The channels of positive streamers thus have a larger plasma density. Positive streamers may move more quickly or more slowly than negative streamers [18]. Numerical simulation is a more useful technique for analyzing the streamer than experimentation because of the short time scale of streamer propagation [19]. Additionally, filamentary structures with sizes smaller than one millimeter are found in streamers. Because of these features, it is very difficult to determine the microscopic properties of streamers through experimental research [20]. Understanding and developing plasma theory has been greatly aided by the simulation [21]. Because ions generate energy more slowly and lose it more easily in collisions, electrons dominate the physics of streamer discharges [22]. There are four different kinds of streamer models: hybrid models, kinetic models, fluid models, and particle models [23]. Streamer discharges are typically simulated using both fluid and kinetic/particle-in-cell models [24]. Generally speaking, kinetic simulations have a far higher computing cost than fluid simulations since they require a larger number of particles and smaller time steps. For the fluid models, particle densities (and energy densities or momentum) vary with time, and input data are pre-calculated transport coefficients

[22]. Dhali and Williams published the first 2D simulation of streamer discharges [25].

In electrode spacing ranging from centimeters and millimeters to meters and larger, the dielectric breakdown (BD) in air under different conditions was studied [1]. The focus of this research is on positive streamers in the atmosphere, which form more readily than negative ones. Positive streamers need an electron source ahead of them because they propagate against the electron drift velocity. These electrons rapidly multiply due to electron-impact ionization when they enter the high-field region surrounding a positive streamer head, causing the streamer channel to lengthen [15].

In this simulation, the positive streamer discharge was simulated using the fluid model [26]. Based on the following 'drift-diffusion' equations for ions and electrons combined with the Poisson's equation, the most popular and useful model to investigate the dynamics of streamers [27]. The COMSOL program is used for the finite element modeling [28]. Starting close to the high-voltage electrode, the streamer moved along the gap's axis to the grounded electrode. This paper's main focus is on the impact of various mechanisms, including impact ionization, attachment, and recombination.

2. Model of Simulation

The cathode directed streamer was modeled in COMSOL Multiphysics using the Plasma module (plas). There is documentation available in [29-31] regarding the plasma module. It performs the function of coupling the heavy species transport, drift diffusion and electrostatics physics into an integrated multi-physics interface to model plasma discharges [32,33].

2.1 Mathematical model

The creation, displacement, and disappearance of charged particles in the form of fluid motion can be described by the movement of charged particles under an electric field [28]. The fluid model is a stiff system of continuous equations dominated by convection that is closely related to Poisson's equation [20]. In order to account for the rates of the physical processes, the formulation yields partial differential equations, or PDEs [34]. The main physical parameters required for simulation in this work are listed in table (1) [35-37]. For electrons, positive ions, and negative ions, the coupled continuity equations are [38]:

$$\frac{\partial n_e}{\partial t} + \nabla . (n_e \mu_e E - D_e \nabla n_e) = \alpha\, n_e |\mu_e E| - \eta\, n_e |\mu_e E| - \beta_{ep} n_e n_p \quad (1)$$

$$\frac{\partial n_p}{\partial t} + \nabla . (n_p \mu_p E - D_p \nabla n_p) = \alpha\, n_e |\mu_e E| - \beta_{ep} n_e n_p - \beta_{pn} n_p n_n \quad (2)$$

$$\frac{\partial n_n}{\partial t} + \nabla . (n_n \mu_n E - D_n \nabla n_n) = \eta\, n_e |\mu_e E| - \beta_{pn} n_p n_n \quad (3)$$

Table (1) The variables and parameters used in the simulations

Properties	Functions
α (cm^{-1})	3500 exp (-1.65x10^5 E^{-1})
η (cm^{-1})	15 exp (-2.5x10^4 E^{-1})
β_{ep} (cm^3/s)	2x10^{-7}
β_{np} (cm^3/s)	2x10^{-7}
DeL (cm^2/s)	1800 The direction of the longitudinal diffusion coefficients
DeT (cm^2/s)	2190 The transverse direction diffusion coefficients
μ_e ($cm^2/V.s$)	2.9x10^5 /P
μ_i ($cm^2/V.s$)	2.6x10^3 /P

These formulas take into consideration diffusion, ionization, attachment, recombination, and reactions in addition to the drift of the charged species in the electric field [39]. These equations are solved using the 'finite element method' (FEM). The formation of electron avalanches in air is explained by equations 1, 2, and 3 [40] and explain how the three charged species are generated, lost, and moving [12]. The quantities associated with negative ions, electrons, and positive ions are denoted by the subscripts *n*, *e*, and *p*, respectively. The charge carrying density is denoted by *n*. Functions μ_n, μ_p and μ_e are respectively the mobility of a negative ion, a positive ion, and an electron, whereas D_n, D_p and D_e are respectively the diffusion coefficient of a negative ion, a positive ion, and an electron. The function denoted by α represents the ionization coefficient of neutral molecules, η denotes the attachment coefficient of electrons and neutral molecules β_{ep} denotes the recombination coefficient of electrons and positive ions, and β_{pn} denotes the recombination coefficient of negative and positive ions. By neglected the effect of the magnetic field [39] in an 'electrostatic field', an internal electric field is produced by the plasma's negative & positive charges. The Poisson equation must be applied in addition to the appropriate potential conditions being set at the anode and cathode to account for the space charge effect [41,42].

$$\nabla^2 V = -(net\ charge\ density)/\varepsilon_0 \quad (4)$$

$$\nabla^2 V = \frac{|e|}{\varepsilon_0}(n_e + n_n - n_p) \quad (5)$$

where e is the electron charge (1.6x10^{-19} C), and ε_0 is the permittivity of vacuum (8.85x10^{-14} F/cm)

2.2 The physical model

To simulate 'the actual application' environment of an insulating medium, two-dimensional axisymmetric (r, z dependent) is often used in case of streamer discharge in atmospheric-pressure air because of its ease of use and low calculation cost. The geometric model which used in this simulation show in Fig. (1), the vertical red line that is broken and runs from top to bottom is the axis of symmetry. The upper boundary is the plane electrode, which is set as 'the cathode', is grounded. The lower boundary is the plane electrode, which is set as 'the anode', is subjected to a positive dc high voltage of 16 kV giving a field of 32 kV/cm, and the remaining boundary is 'an open boundary'.

In Fig. (1), the boundary conditions for the various PDEs are summarized. A Gaussian patch of neutral plasma is placed on the electrode tip to start the streamer. A Gaussian form with 1/e radii of 0.027 and 0.021 cm in the z and r axes, respectively, and a peak density of 10^{14} cm^{-3}. A uniform, 'neutral ionization density' of 10^{8} cm^{-3} was placed at 0.0001 cm from the anode electrode.

$$n_e(r,z)|_{t=0} = n_p(r,z)|_{t=0} = 10^8 + n_0 exp[-(\frac{z-z_0}{s_z})^2 - (\frac{r-r_0}{s_r})^2] \quad (6)$$

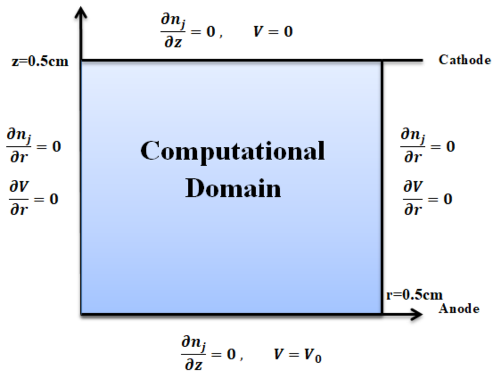
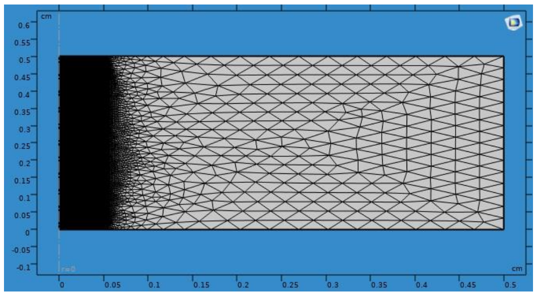
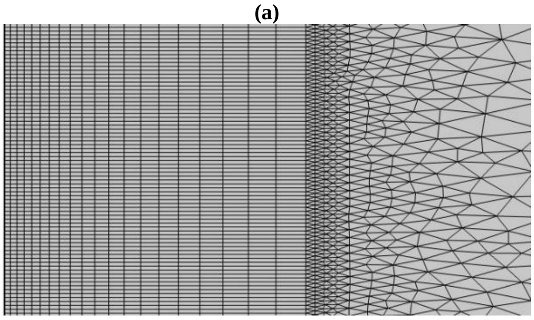


Fig. (1) The configuration of the discharge simulation

The important phase of any simulations is the mesh adjustment [43]. The finite element method is usually used in the simulation solution of the 'electric field' problems. The basic idea of FEM is to partition the study area into finite elements, often known as meshes. During discharge; the charge particles density and the electric field will change inside the streamer channel more than at the electrode edge. The model required a high resolution mesh in the regions which have high electric field consequently helps in lowering the simulation time of the streamer discharge. Given that the correctness of the results that describe the generation and propagation of a streamer in the atmospheric air depends on the selection of the mesh element shape. The simulation needs very thin elements near the axisymmetric and any region which have high ion densities. Although using such thin meshes results in results graphs with finer resolution, the simulation takes longer to complete [44]. Figure (2a) the simulation region is divided into several fine grids, with the most suitable grid structure being applied close to the symmetry line. In areas distant from the symmetry axis, we employ a coarse network structure, which significantly increases the precision of numerical computations [45]. The mesh zoom and the precision of some mesh entanglement in this model are shown in Fig. (2b).



(a)



(b)

Fig. (2) 2D axial symmetric diagrams (a) mesh map area (b) Zoom of the mesh map area around the symmetric line

3. Results and Discussions

The applied voltage for the majority of the computations we show here was 16 kV. The results of the positive streamer simulation are as follows:

3.1 Electron density

In order to simulate the growth and course of the streamer within the simulation, we used the electron density because it's the criterion of the initiation of the streamer discharge in air; the range of the electron density criterion is 10^{13}-10^{14} cm^{-3} [8,40,47-50]. A continuity equation must be solved in order to determine the electrons' number density [46]. We used the colors red to indicate a high electron density and blue to represent a low electron density in order to express the electron density in the solution zone. The two primary steps of the streamer initiation and propagation process are visible in Fig. (3).

Once the initial avalanche head reaches the anode, positive flow begins from the anode to the cathode. While a streamer is developing, its characteristics alter because (i) the increasing voltage drop across the streamer channel leads to a decrease in the potential of the streamer head, (ii) the local electric field increases when the streamer head approaches the opposite electrode, and (iii) The high voltage electrode's seed electron supply is crucial for a streamer to propagate continuously.

The plasma channel had strong conductivity throughout the gap when the streamer reached the cathode. The streamer required 15 ns to start and spread throughout the whole air gap. The amount of time that the software keeps the simulation results is known as the simulation step size. The streamer's dynamic process becomes more accurate with fewer time intervals, but the computer system needs more memory to support them. The streamer creation process may lose a lot of detail if the simulation step is too lengthy. Table (2) displays the data for the positive streamer.

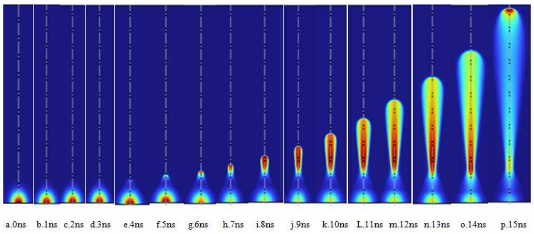
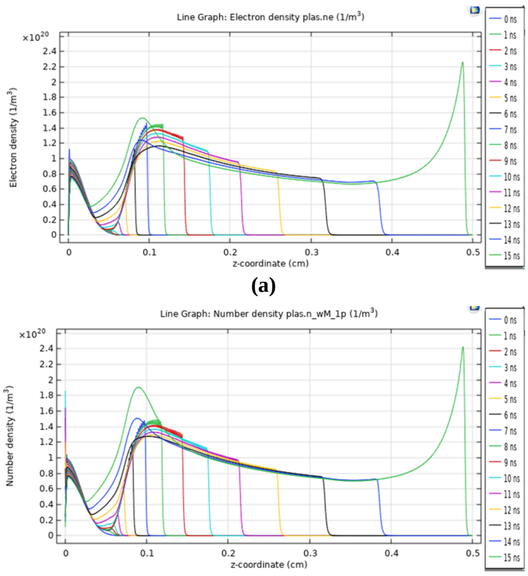


Fig. (3) Profiles of electron concentrations over time along the symmetry axis in 0.5 cm air gap

Figure (4) shows the line graph of different particles species distribution along axial direction (the negative ions, positive ions and electrons density) which is plotted at several points during the streamer simulation against the symmetry axis.



(a)

(b)

Fig. (4) Spatial distributions of (a) the electron number density and (b) ion number density along their respective axis of symmetry during many time instants during the 'streamer propagation'

The streamer initiates at t = 1-5 ns, as seen in this figure. The electron densities ~10^{13} cm^{-3} and positive-ion densities are >10^{14} cm^{-3} while the negative-ion density is <10^{13} cm^{-3}. Between t=6-10 ns the streamer propagates, due to the slow progress of the streamer, near the streamer head, the electron density is around ~10^{14} cm^{-3}, but it decreases to <10^{14} cm^{-3} along the channel; this fall results from three processes: (1) recombination with positive ions, (2) production of negative ions from attachment, and (3) electron drift to the anode. Near the streamer head, the positive-ion density is >10^{14} cm^{-3}, however, as one move down the channel, it falls to <10^{14} cm^{-3}; this decrease is caused by recombination with electrons and negative ions. Streamer termination, between t=11-15 ns, the electron and positive ion densities are extremely low throughout the channel at 15 ns, with the exception of a dramatic spike at 'the streamer head'. The cathode is nearly reached by the streamer. The negative ion density is higher than the electron density everywhere in the channel, which is equal to the relatively flat distribution of positive ions. Positive ions are found in greater numerical density than electrons and negative ions everywhere. The line diagram also shows that the electron density concentrations range in magnitude from 10^{13}-10^{14} cm^{-3}.

3.2 Electric field distribution

When an intense enough external electric field is introduced; the quantity of charged particles grows dramatically. Electrons and negative (positive) ions travel in the opposite (same) direction of the generated electric field lines following the creation of charged particles. The electric field distribution suddenly shifts as the streamer develops due to the increased quantity of charged particles. At the same time, the space charge distribution varies as a result of charged particles moving around in the electric field. Thus, during the growth of the streamer, interactions between the motion of charged particles and the electric field produced by the space charge have a considerable impact on its development. When the streamer discharge develops and propagates in air gap. The Poisson equation is used to compute the electric potential and the electric field in order to ascertain the distribution of the electric field in the configuration of the electrodes. A numerical method for resolving problems of this kind is the finite element approach. The electric field value change on the air gap length; this is causes two main regions: The space charge in the positive streamer causes a large electric field at its head, and the channel of the streamer symbolizes plasma, which is known to be empty due to high electric fields. Figure (5) displays the electric field distribution along the symmetry axis, and it is possible to see a highly deformed area close to the anode. It's interesting to note that as one approaches the cathode plate, the electric field starts to increase & reduces near the anode plate.

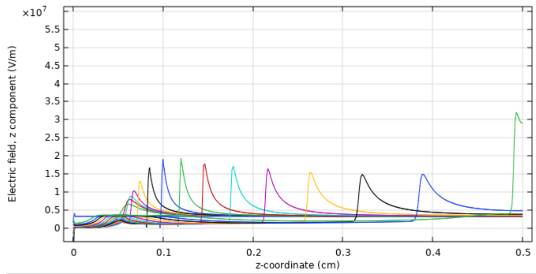


Fig. (5) The evolution of electric field with time for various time steps along the streamer axis

Figure (5) corresponds to the electron density profiles shown in Fig. (4) and shows the line diagram for the time evolution of the electric field distribution V/m along the z-axis from 1 ns to 15 ns. Based on the findings, the electric field strength at the streamer's head varies from 6.58x10^4 V/cm and 3.21x10^5 V/cm.

The electric field values along the air gap during the start and propagation of the streamer were displayed in table (2). It was found that as the streamer discharge developed, the field strength at the streamer channel remained largely constant. Since the ionization of background molecules and the migration of charged carriers under the strong electric field are what keep the closely linked processes of streamer discharge propagation stable, the electric field strength is normally constant. The interaction between the streamer head and the grounded electrode is responsible for the increase in electric field that occurs as the streamer approaches the cathode. At time t = 15 ns, the positive streamer near the flat electrode causes a sharp increase in the electric field at the head of the streamer, which eventually leads to a breakdown. This reflects the electrostatic interaction between the plasma channel with a certain electrical potential and the grounded electrode.

Table (2) Information of the positive streamer which obtained from the simulation

Time Evolution (ns)	Distance (cm)	Velocity $x10^7$ (cm/s)	Electron Density $x10^{13}$ (cm^{-3})	Electric Field $x10^5$ (V/cm)
1	0.05	5.00	9.80	0.658
2	0.055	2.80	9.56	0.790
3	0.06	2.00	9.44	0.874
4	0.065	1.60	9.28	1.03
5	0.07	1.40	9.08	1.28
6	0.083	1.38	11.20	1.67
7	0.098	1.39	14.70	1.90
8	0.118	1.48	14.50	1.92
9	0.144	1.60	13.80	1.77
10	0.176	1.80	13.30	1.71
11	0.215	1.95	12.80	1.64
12	0.262	2.18	12.30	1.54
13	0.319	2.45	11.70	1.49
14	0.386	2.76	12.40	1.48
15	0.492	3.28	22.60	3.21

4. Conclusions

This paper presented a simulation method and analysis of the stages of discharges in atmospheric air under applied positive dc voltage for a plane-plane electrode with 0.5cm gap. The minimum applied voltage required for propagation of positive streamer discharge in air was 16 kV. From the results; we can see the peak electric field at the streamer head, velocity, the electrons density, the ions density, head radius and channel radius are change with the time evolution. The electric field strength is enhanced as the streamer approaches the cathode, and finally, a channel is created across the discharge region in which the electron density is about 10^{13}-10^{14} cm^{-3}. The negative ion density in the streamer channel is higher than the positive ion density in the streamer channel while the positive ion density in the streamer head is higher than the negative ion density in the streamer head.

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